arXiv:hep-ph/0612366v1 29 Dec 2006

Charge density of a positively charged vector boson may be negative

V.V.Flambaum* and M.Yu.Kuchiev†

School of Physics, University of New South Wales, Sydney 2052, Australia

(Dated: January 1, 2007)

The charge density of vector particles, for example $W^{\pm}$, may change sign. The effect manifests itself even for a free propagation; when the energy of the W-boson satisfies $\varepsilon > \sqrt{2}m$ and the standing-wave is considered the charge density oscillates in space. The charge density of W also changes sign in a close vicinity of a Coulomb center. The dependence of this effect on the g-factor for an arbitrary vector boson, for example ρ-meson , is discussed. An origin of this surprising effect is traced to the electric quadrupole moment and spin-orbit interaction of vector particles. Their contributions to the current have a polarization nature. The charge density of this current, $\rho_{\,\rm Pol} = -\nabla\cdot \mathrm{P}$, where P is an effective polarization vector that depends on the quadrupole moment and spin-orbit interaction, oscillates in space, producing zero contribution to the total charge.

PACS numbers: 12.15.Ji, 12.15.Lk, 12.20.Ds

We show that the charge density of vector bosons, W bosons in particular, can change sign. The effect manifests itself either for sufficiently high energy of the W-boson, or for sufficiently strong attractive potential. The origin of this effect is related to the electric quadrupole moment and spin-orbit interaction of vector mesons.

The Proca theory [1] presumes that the magnetic g-factor of vector bosons is $g = 1$. The Corben-Schwinger [2] formalism allows one to describe vector bosons with an arbitrary value of the g-factor. In particular, it is applicable for the case of W bosons in the Standard Model, which have $g = 2$, see e. g. [3]. A close correspondence between the Corben-Schwinger formalism and the gauge theory was discussed in [4], its relation to the Standard Model was discussed in [5, 6].

Following [2] let us describe a vector boson propagating in an electromagnetic field by the Lagrangian

$$\begin{aligned}\mathcal{L} \;&=\; -\frac{1}{2}\, W_{\mu\nu}W^{\mu\nu} + m^2\, W^+_\mu W^\mu + \\ &+ i\, e\,(g-1)F^{\mu\nu}\, W^+_\mu W_\nu \;. \qquad (1)\end{aligned}$$

Here m is the mass of a vector boson, $\nabla_\mu = \partial_\mu + ieA_\mu$, $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$, $W_{\mu\nu} = \nabla_\mu W_\nu - \nabla_\nu W_\mu$ and the magnetic g-factor is arbitrary. The Lagrangian (1) gives the following wave equations [2]

$$\left(\nabla^2 + m^2\right) W^\mu + ieg\, F^{\mu\nu}\, W_\nu - \nabla^\mu\nabla^\nu\, W_\nu = 0 \;. \qquad (2)$$

Taking a covariant derivative in Eq.(2) one finds also [2]

$$m^2\nabla_\mu W^\mu + ie(g-1)\, J_\mu W^\mu - ie\frac{g-2}{2}F^{\mu\nu}W_{\mu\nu} = 0 \;, \qquad (3)$$

where $J^\mu = \partial_\nu F^{\nu\mu}$ is the external current, which creates the external field $F^{\nu\mu}$. Eq.(3) guarantees that among four components of the vector W_μ only three are independent, precisely how it should be for a massive particle. From Eq.(1) one also derives the current of vector bosons j_μ [2]

$$j_{\,\mu} = ie\Big(W^{+\nu}W_{\mu\nu} + (g-1)\partial^\nu(W^+_\mu W_\nu) - c.c.\Big) \;. \qquad (4)$$

Here $c.c.$ refers to two complex conjugated terms. Let us start from the simplest case of a free motion, when Eq.(4) gives the following charge density for vector particles

$$\begin{aligned}\rho \equiv j_{\,0} = e\Big(&2\varepsilon(\boldsymbol{W}^+\!\cdot\boldsymbol{W} + (g-1)W^+_0 W_0) \\ &+ ig(\boldsymbol{W}^+\!\cdot\boldsymbol{\nabla}W_0 - \boldsymbol{W}\!\cdot\boldsymbol{\nabla}W^+_0)\Big) \;. \qquad (5)\end{aligned}$$

Eq. (3) gives $W_0 = -i(\boldsymbol{\nabla}\cdot\boldsymbol{W})/\varepsilon$. For the plane wave $\boldsymbol{W} = \boldsymbol{C}\exp(i\boldsymbol{p}\!\cdot\!\boldsymbol{r})$ one observes a conventional, positively defined charged density

$$\rho = (2e/\varepsilon)\big(|\,C\,|^2\varepsilon^2 - |\,\boldsymbol{C}\cdot\boldsymbol{p}\,|^2\big) \;, \qquad (6)$$

where $p = (\varepsilon^2 - m^2)^{1/2}$ is the momentum. However, for the standing wave $\boldsymbol{W} = \boldsymbol{C}\sin(\boldsymbol{p}\cdot\boldsymbol{r})$ an unusual effect takes place, the sign of the charge density is not necessarily fixed. The charge density in this case reads

$$\begin{aligned}\rho \;&=\; \frac{2e}{\varepsilon}\Big((g-1)|\,\boldsymbol{C}\cdot\boldsymbol{p}\,|^2 \\ &-\; \big(\,(2g-1)|\,\boldsymbol{C}\cdot\boldsymbol{p}\,|^2 - |\,C\,|^2\varepsilon^2\,\big)\sin^2(\boldsymbol{p}\cdot\boldsymbol{r})\Big). \qquad (7)\end{aligned}$$

For the longitudinal polarization, $|\,\boldsymbol{C}\cdot\boldsymbol{p}\,| = |\,C\,|p$, the density is

$$\rho = \frac{2e|C|^2}{\varepsilon}\Big((g-1)p^2 - \big((2g-1)p^2 - \varepsilon^2\big)\sin^2(\boldsymbol{p}\!\cdot\!\boldsymbol{r})\Big). \qquad (8)$$

We see that for $g > 1$ and energy $\varepsilon > m\sqrt{g/(g-1)}$ the charge density may take negative values. In the case of the W-boson, $g = 2$, the change of sign appears for the energies $\varepsilon > \sqrt{2}m$. The minimum of density corresponds

*Email:flambaum@phys.unsw.edu.au
†Email:kuchiev@phys.unsw.edu.au

to $\sin^2(\boldsymbol{p}\cdot\boldsymbol{r}) = 1$. In the Proca case $g = 1$ the sign of the charge density is fixed. However, for $g < 1$ the charge density may be negative again. The minimum of the density in this case corresponds to $\sin(\boldsymbol{p}\cdot\boldsymbol{r}) = 0$. As we will show below this behavior is explained by the contribution of the electric quadrupole moment of the vector particle, $Q \propto (g-1)e/m^2$.

The motion of the W-boson (g=2) in the attractive Coulomb potential $U = -Z\alpha/r$ gives another example. Take the most interesting case of the total angular momentum $j = 0$. Then $\mathbf{W}(\mathbf{r}) = \mathbf{n}\, v(r)$, where $\mathbf{n} = \mathbf{r}/r$ and $v(r)$ is the radial wave function [2, 5]. Eq. (4) gives the density of charge for this state,

$$\rho = 2e\Big((\varepsilon - U)(v^2 + w^2) + 2v\frac{dw}{dr}\Big)\ , \tag{9}$$

where $w(r) = (v'(r) + 2v(r)/r)/(\varepsilon - U)$ [2, 5]. At large distance we may neglect the potential, and the wave equation has usual spherical-wave solution $v(r) \approx \sin(p\, r + \delta)/r$ or $\exp(-\kappa r)/r$. Again, the change of sign appears for energies $\varepsilon > \sqrt{2}m$. The result looks even more interesting for a charge density in a vicinity of the Coulomb center. The most singular term in the Coulomb solution for W-boson is $v(r) \sim r^{\gamma-3/2}$ where $\gamma = (1/4 - Z^2\alpha^2)^{1/2}$ [2, 5] . The charge density in this case is $\rho \propto -e\, r^{2\gamma-4}$ [2, 5]. Thus, the charge in a vicinity of the Coulomb center has the "wrong" sign. Moreover, the integral of the charge density is divergent and this wrong-sign charge is infinite. In [5] we showed that the Coulomb problem for W-boson is saved by the fermion vacuum polarization which produces the impenetrable potential barrier near the origin, $U_{eff} \sim Z^2\alpha^3/m^3r^4$ [7]. As a result, the W-boson charge density decreases exponentially and vanishes at $r = 0$, which makes the charge with wrong-sign near the origin finite.

To make a physical nature of this phenomenon more transparent consider the nonrelativistic approximation for the Hamiltonian that describes a vector particle in a static electric field (this Hamiltonian for $g = 2$ was derived in [5])

$$\begin{aligned} H_{ij} \ =\ & \left(\frac{\boldsymbol{p}^2}{2m} + U\right)\delta_{ij} - \frac{\boldsymbol{p}^4}{8m^3}\,\delta_{ij} \\ & -\frac{g-1}{2m^2}\left(\,\boldsymbol{F}\cdot(\boldsymbol{p}\times\boldsymbol{S}_{ij}) - \nabla_i\nabla_j\, U\,\right)\ , \end{aligned} \tag{10}$$

Here $i, j = 1, 2, 3$ label components of three-vectors, the nonrelativistic Schredinger equation in this notation reads $H_{ij}\Phi_j = E\Phi_i$, $\boldsymbol{S}$ is the spin, which operates on a vector $\boldsymbol{V}$ according to $\boldsymbol{S}_{ij}V_j = -i\epsilon_{ijk}V_k$, $U = eA_0$ and $\mathbf{F} = -\boldsymbol{\nabla}U$ are the potential energy and the force produced by the field. The first and second terms in Eq.(10) describe the basic nonrelativistic approximation and the relativistic correction to the mass respectively, the term with the spin gives the spin-orbit interaction. The last term in Eq.(10) includes two contributions: the contact term $\propto \Delta U\delta_{ij}/3$ and the quadrupole moment term $\propto \nabla_i\nabla_j U - \Delta U\,\delta_{ij}/3$. The corresponding density of the electric quadrupole moment equals

$$Q_{ij} = (g-1)\,\frac{e}{m^2}\left(\,3\boldsymbol{\Phi}_i^*\boldsymbol{\Phi}_j - \delta_{ij}|\boldsymbol{\Phi}|^2\right)\ , \tag{11}$$

where

$$\boldsymbol{\Phi} \simeq \boldsymbol{W} + \boldsymbol{\nabla}\,(\boldsymbol{\nabla}\cdot\boldsymbol{W})/2m^2\ , \tag{12}$$

is the non-relativistic wave function introduced in [5].

Calculating a variation of the matrix element $\langle\Phi|H|\Phi\rangle$ of the Hamiltonian Eq.(10) with respect to the potential we find the charge density $\rho_{\rm nr}$ of a vector particle in the nonrelativistic approximation:

$$\rho_{\rm nr} \ =\ \rho_{\rm C} + \rho_{\rm S} + \rho_{\rm Q}\ , \tag{13}$$

$$\rho_{\rm C} \ =\ e\,\boldsymbol{\Phi}_i^*\boldsymbol{\Phi}_i\ , \tag{14}$$

$$\rho_{\rm S} \ =\ (g-1)\,\frac{e}{2m^2}\,(\,\nabla_i\boldsymbol{\Phi}_i^*\nabla_j\boldsymbol{\Phi}_j - \nabla_j\boldsymbol{\Phi}_i^*\nabla_i\boldsymbol{\Phi}_j\,), \tag{15}$$

$$\rho_{\rm Q} \ =\ (g-1)\,\frac{e}{2m^2}\,\nabla_i\nabla_j\,(\boldsymbol{\Phi}_i^*\boldsymbol{\Phi}_j)\ . \tag{16}$$

The term $\rho_{\rm C}$ here can be interpreted as a conventional nonrelativistic distribution of charge. The next term $\rho_{\rm S}$ originates from the spin-orbit interaction in Eq.(10). The term $\rho_{\rm Q}$ comes from the last term in the Hamiltonian (10), which describes the quadrupole and contact interactions of the boson. The spin-orbit and quadrupole terms give zero contribution to the total charge because each one of them can be written as a divergence. For the quadrupole term $\rho_{\rm Q}^W$ this is evident from its definition. The spin-orbit term can be rewritten in such a way, $\rho_{\rm S}^W \propto \nabla_i\big(\boldsymbol{\Phi}_i^*\nabla_j\boldsymbol{\Phi}_j - \boldsymbol{\Phi}_j^*\nabla_j\boldsymbol{\Phi}_i)\Big)$, as to make it clear that it is a divergence as well. Thus, the sum $\rho_{\rm S} + \rho_{\rm Q}$ may be presented as a divergence of a vector,

$$\rho_{\rm S} + \rho_{\rm Q} \equiv\ \rho_{\,\rm Pol}\ = -\boldsymbol{\nabla}\cdot\boldsymbol{P}\,. \tag{17}$$

Therefore, it can be looked at as a charge density $\rho_{\,\rm Pol}$ induced by an effective polarization which is described by the polarization vector $\boldsymbol{P}$ [8]. This vector can be identically rewritten in the following simple form

$$\boldsymbol{P} = -(g-1)\,\frac{e}{m^2}\,{\rm Re}\left(\,\boldsymbol{\Phi}^*(\boldsymbol{\nabla}\cdot\boldsymbol{\Phi})\,\right) \tag{18}$$

Eq.(17) explicitly shows that $\rho_{\,\rm Pol}$ does not contribute to the total charge of the vector boson, $\int\rho_{\,\rm Pol}d^3r = 0$. Consequently, $\rho_{\,\rm Pol}$ inevitably oscillates, changing sign. Same conclusion holds separately for $\rho_{\rm S}$ and $\rho_{\rm Q}$.

In the low-energy region $\rho_{\,\rm Pol}$ is smaller than the conventional charge density $\rho_{\rm C}$, which has a definite sign. However, with increase of momentum $|\rho_{\,\rm Pol}/\rho_{\rm C}|$ is growing as p^2/m^2 or $1/r^2m^2$ since it depends on derivatives of the wave function. For sufficiently high energy (or large attractive potential) $\rho_{\,\rm Pol}$ may prevail, forcing the total charge density ρ to change its sign as well. However, to make this argument rock solid for high energies, one has to consider a fully relativistic treatment, which is presented below, see Eqs.(27)-(30).

To clarify the meaning of Eqs.(13)-(18) let us consider specific cases for a low-energy vector particle. For the plane wave $\boldsymbol{W} = \boldsymbol{C}\exp(i\boldsymbol{p}\cdot\boldsymbol{r})$ the spin-orbit and quadrupole contributions to the charge density vanish, $\rho_{\rm Q} = \rho_{\rm S} = 0$, and the density $\rho = \rho_{\rm C} = const$, in agreement with Eq. (6). For the standing wave $\boldsymbol{W} = \boldsymbol{C}\sin(\boldsymbol{p}\cdot\boldsymbol{r})$ we obtain the positive non-relativistic charge density, zero spin density and oscillating quadrupole density

$$\rho_{\rm C} = e(|C|^2 - |\boldsymbol{C}\cdot\boldsymbol{p}/m|^2)\sin^2(\boldsymbol{p}\cdot\boldsymbol{r}) \ , \qquad (19)$$

$$\rho_{\rm S} = 0 \ , \qquad (20)$$

$$\rho_{\rm Q} = e(g-1)|\boldsymbol{C}\cdot\boldsymbol{p}/m|^2\cos(2\boldsymbol{p}\cdot\boldsymbol{r}) \ . \qquad (21)$$

As expected, the quadrupole density increases with p^2. The total density agrees with Eq.(7) for $\varepsilon \approx m$.

Finally, consider a vector particle in the attractive Coulomb field in the state $2p_0$ ($j = 0,\ l = 1$). In this case

$$\boldsymbol{\Phi} = C\boldsymbol{r}\exp(-kr/2) \ , \qquad (22)$$

$$\rho_{\rm C} = eC^2r^2\exp(-kr) \ , \qquad (23)$$

$$\rho_{\rm S} = (g-1)eC^2(3-kr)\exp(-kr)/m^2 \ , \qquad (24)$$

$$\rho_{\rm Q} = (g-1)eC^2(6-4kr+k^2r^2/2)\exp(-kr)/m^2 . \qquad (25)$$

Here $k = Z\alpha m$, while C describes the normalization of the wave function. We see that the spin density and the quadrupole density are enhanced by the factor $\propto 1/m^2r^2$ at small distances in comparison with the non-relativistic charge density $\rho_{\rm C} \propto r^2$. Also, $\rho_{\rm S}$ and $\rho_{\rm Q}$ have the radial oscillations and vanish after the radial integration, $\int \rho_Q\, r^2 dr = \int \rho_S\, r^2 dr = 0$.

Let us discuss now the relativistic case. For simplicity consider propagation of the W-boson ($g = 2$) in a static external field $U = eA_0$, $\mathbf{A} = 0$. In the region, where the external current is absent, $J_\mu = 0$, Eq.(3) gives

$$w \equiv i\,W_0 = \boldsymbol{\nabla}\cdot\boldsymbol{W}/(\varepsilon - U) \ , \qquad (26)$$

and the charge density extracted from Eq.(4) may be presented in the following form:

$$\rho = \rho_{\rm P} + \rho_{CS} \ , \qquad (27)$$

$$\rho_{\rm P} = 2e\Big((\varepsilon - U)\,\boldsymbol{W}^{+}\!\cdot\boldsymbol{W} + {\rm Re}\,(\boldsymbol{W}^{+}\!\cdot\boldsymbol{\nabla}w)\Big) \ , \qquad (28)$$

$$\rho_{\rm CS} = -\boldsymbol{\nabla}\cdot\boldsymbol{P}_{\rm CS} \ , \qquad (29)$$

$$\boldsymbol{P}_{\rm CS} = -2e\,{\rm Re}\,(\boldsymbol{W}^{+}w) \ . \qquad (30)$$

Here $\rho_{\rm P}$ is related to the first two terms in the Lagrangian Eq.(1) which were introduced by Proca [1]. The density $\rho_{\rm CS}$ originates from the last term in Eq.(1), which was firstly introduced by Corben and Schwninger [2] and is also present in the Standard Model (with $g = 2$) [5].

The term $\rho_{\rm P}$ has a conventional, definite sign in the nonrelativistic approximation, when $\rho_{\rm P} \simeq \rho_{\rm C}/(2m)$, where $\rho_{\rm C}$ is defined in Eq.(14). The factor $2m$ here accounts for the fact that we use a conventional notation, in which the normalization conditions for the relativistic and nonrelativistic wave functions differ by precisely this factor. This term also keeps a conventional sign in the region of space where the potential is weak, $|U \ll \varepsilon$. The higher the energy, the wider this region is. In the ultrarelativistic limit $\varepsilon \to \infty$ the term $\rho_{\rm P}$ keeps definite sign almost everywhere, except close vicinities of any Coulomb center. However, since it is known that at in the vicinity of the Coulomb center the the W-boson charge density decreases [5], the small regions near the Coulomb centers give small contribution to the total charge. Thus, for a wide range of energies and vast areas of space the term $\rho_{\rm P}$ in the charge density reveals conventional behavior.

In contrast, the charge density $\rho_{\rm CS}$ in Eq.(29) is a divergence. As a result, it gives zero contribution to the total charge $\int \rho_{\rm CS}\, d^3r = 0$, and inevitably oscillates in space, showing the variation of sign. In the nonrelativistic approximation it is reduced to the polarization charge density defined in Eq.(17), $\rho_{\rm CS} \simeq \rho_{\rm Pol}/(2m)$. The higher the energy the bigger are the spacial derivatives in Eqs.(29),(30), forcing $\rho_{\rm CS}$ to grow with increase of energy. In contrast, $\rho_{\rm P}$ cannot show similar growth due to a restriction related to the total charge of the W boson, which equals the integral $e = \int \rho_{\rm P}\, d^3r$, making large values of $\rho_{\rm P}$ impossible. We conclude that for high energies the term $\rho_{\rm CS}$ prevails in the charge density. It is inevitable therefore that the total charge density of W bosons changes its sing, revealing a "wrong" sign in some areas of space.

In summary, we verified that the charge density of vector particles can have wrong sign; for example it can be negative for the W^+ boson. The effect is related to the electric quadrupole moment and spin-orbit interaction of the vector boson which are proportional to $(g-1)$ and originate from the last term in the Lagrangian Eq.(1), which was firstly introduced by Corben and Schwninger [2] and is also present in the Standard Model (with $g = 2$) [5].

This work was supported by the Australian Research Council.

[1] A. Proca, Compt.Rend. **202**, 1490 (1936).
[2] H. C. Corben and J. Schwinger, Phys.Rev. **58**, 953 (1940).
[3] S. Weinberg, *The Quantum Theory of Fields, Vol. 2: Modern Applications.* Cambridge University Press, 2001.
[4] J. Schwinger Rev. Mod. Phys. **36**, 609 (1964).
[5] M.Yu. Kuchiev and V.V. Flambaum. Mod.Phys.Lett.A **21**, 781 (2006); Phys. Rev. D **73**, 093009 (2006).
[6] V.V. Flambaum and M.Yu. Kuchiev. When renormal-

izability is not sufficient: Coulomb problem for vector bosons, hep-ph/0609194.

[7] This barrier exists for S=1 and does not exist for spin S=0 and S=1/2 particles (where the effect of the vacuum polarization becomes significant only at exponentially small distance $r \sim \exp(-1/\alpha)m^{-1}$). Note that this phenomenon also exists for the repulsive Colomb potential $U = Z\alpha/r$. Indeed, the most singular terms in the wave equation (2) produce an effective potential $\sim -Z^2\alpha^2/mr^2$ [5]. It creates the attraction and divergencies for any sign of Z. The fermion vacuum polarization produces the repulsive barrier $\sim Z^2\alpha^3/m^3r^4$ for any sign of Z too. This phenomenon for the vector bosons in non-Abelian theories $SU(2)$ and the Standard model $U(1) \times SU(2)$ is considered in [6].

[8] One may compare this with a simpler case. Any particle has a small electric dipole moment d. Density of the electric dipole moment, $\mathbf{P}(\mathbf{r}) = \psi^\dagger \hat{\mathbf{d}}\psi$, produces an additional contribition to the charge density, $\rho_d = -\nabla \cdot \mathbf{P}(\mathbf{r})$.